CS107: Dataflow Analysis

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Dataflow Analysis

Dataflow analysis computes value-flow information at each program point in the control-flow graph.

- Intra-procedural: analyses that work on a single function, and whose results are only valid for that function.
- Inter-procedural: analyses that work on multiple functions with function calls and returns.
  - Higher-order program analysis where functions are permitted as first-class values.

A First Example: Available Expressions

The following C program fragment sets `r` to x^y for $y > 0$.

```
1     int y1 = 1;
2     int r = x;
3     while (y1 != y) {
4       int t = y1 * 2;
5       if (t <= y) {
6         r = r * r;
7         y1 = y1 * 2;
8       } else {
9         r = r * x;
10        y1 = y1 + 1;
11      }
12    }
```

Example: `x=2` and `y=5`, then `r=2^5=32` at the end of the program.

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How can it be (slightly) optimized?

CSE

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3     while (y1 != y) {
4       int t = y1 * 2;
5       if (t <= y) {
6         r = r * r;
7         y1 = y1 * 2; // y1 * 2 can be replaced by t
8       } else {
9         r = r * x;
10        y1 = y1 + 1;
11      }
12    }
```

How can it be (slightly) optimized?

Available Expressions

Why is the previous optimization valid?

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- At line 7, where expression `y1*2` appears for the second time, it is **available**.
- That is, no matter how we reach line 7, `y1*2` will have been computed previously at line 4.
- The computation of line 4 is still valid at line 7 because no redefinition of `y1` appears between those two points.

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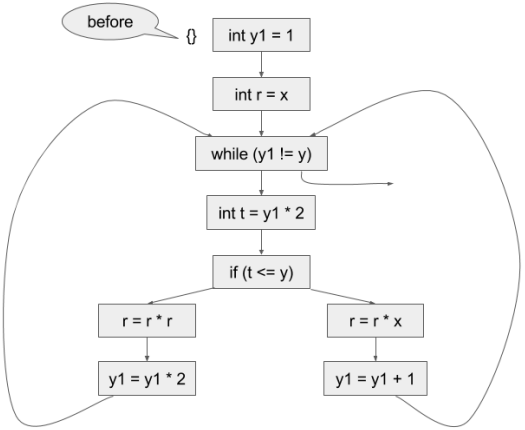
Generally speaking, we can define for every program point the set of **available expressions**, which is the set of all *nontrivial expressions* whose value has already been computed at that point.

Available Expressions



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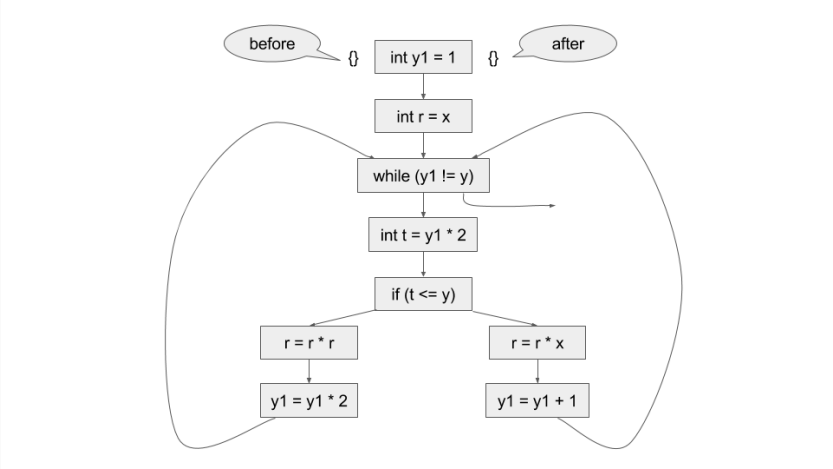

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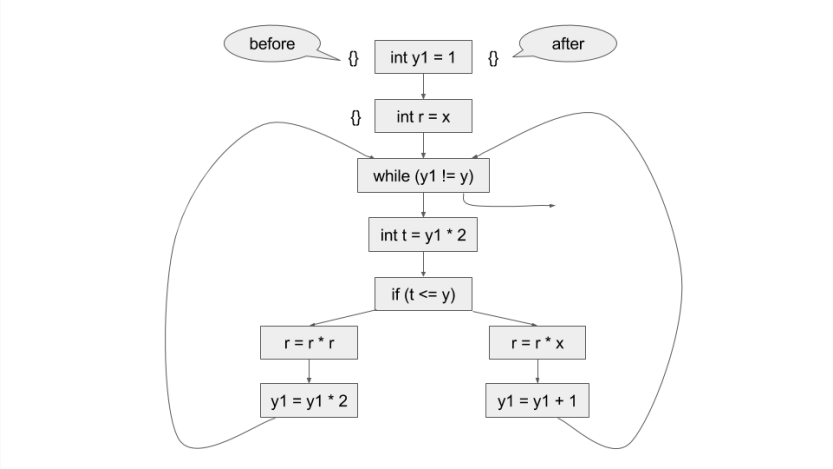

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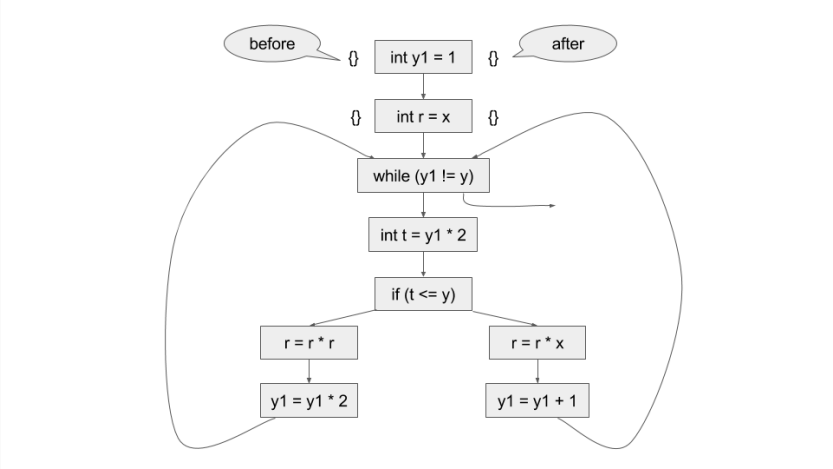

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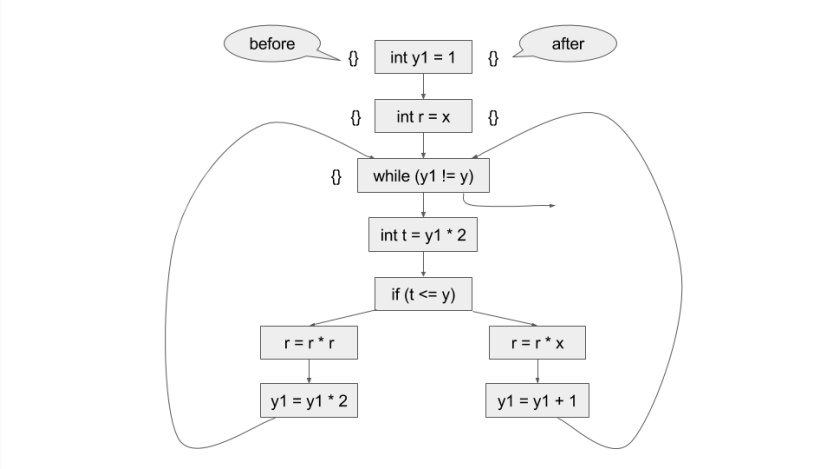

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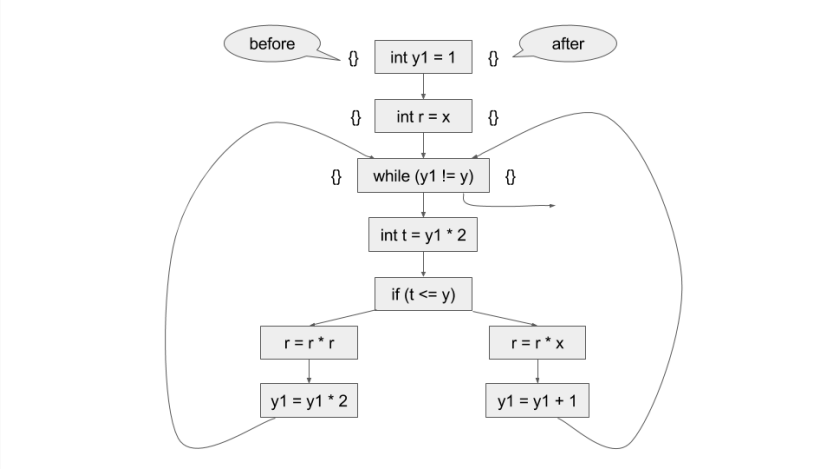

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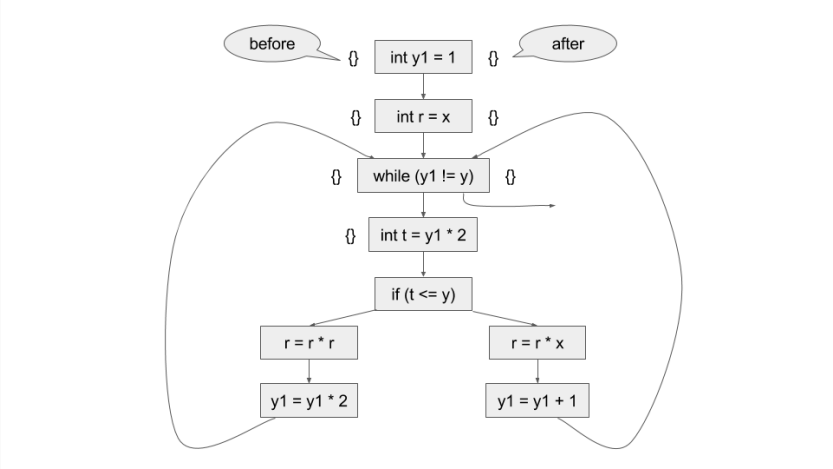

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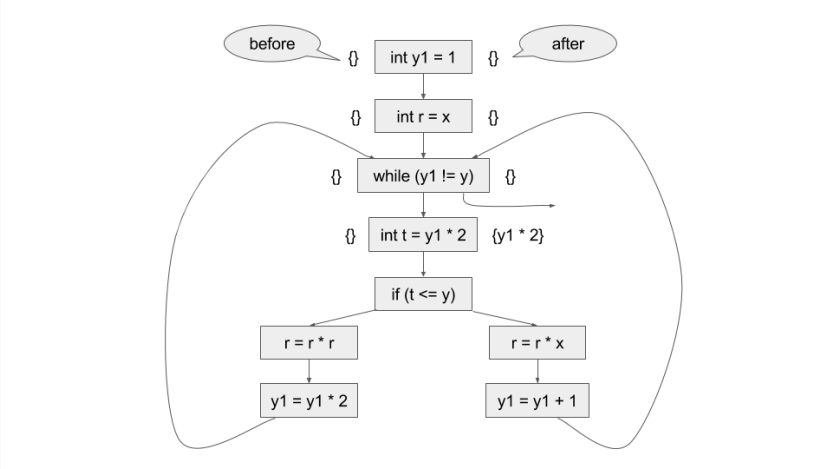

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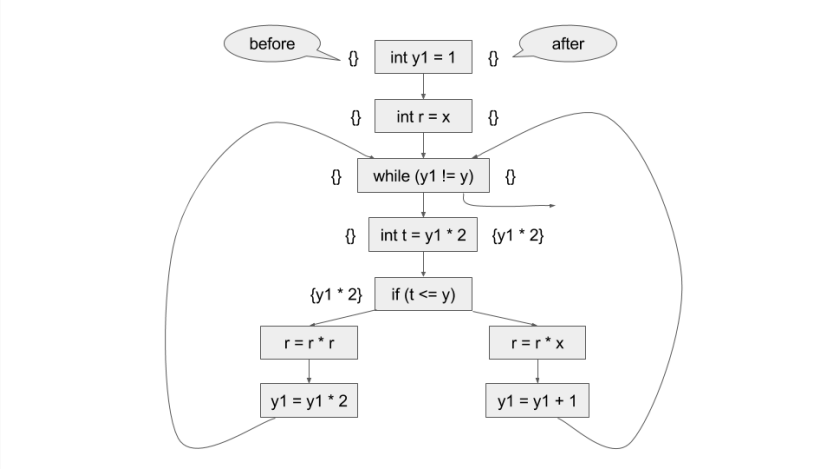

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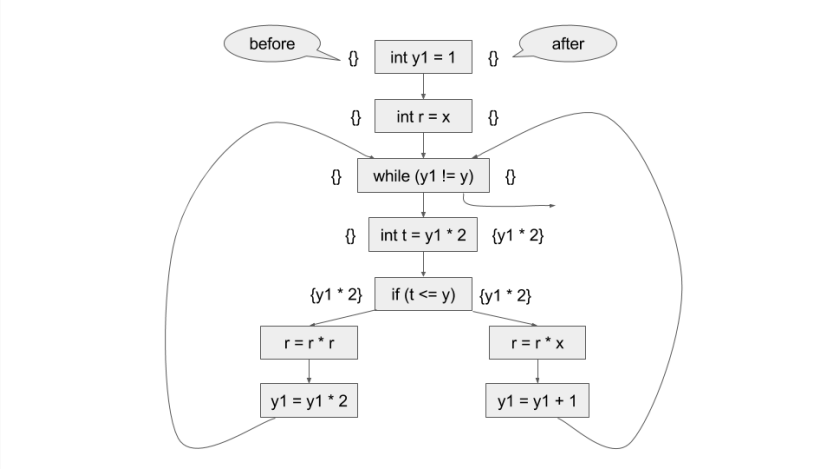

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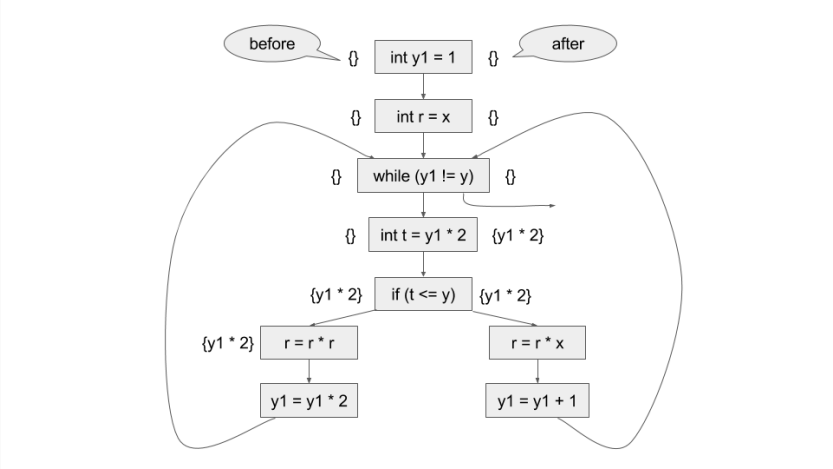

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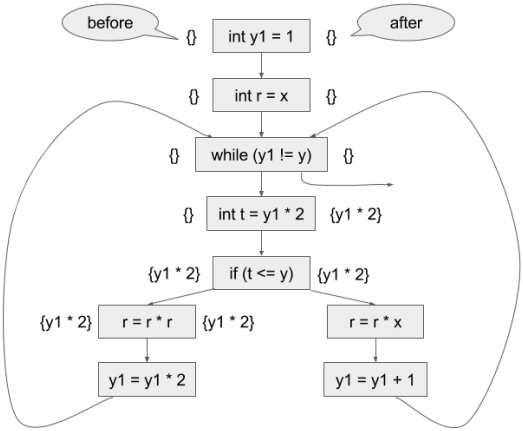

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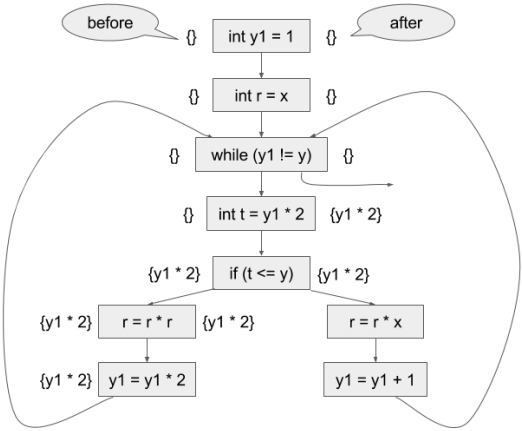

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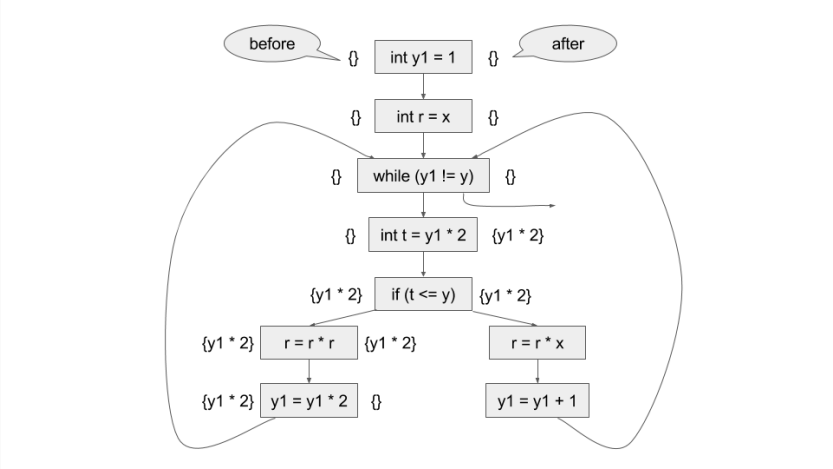

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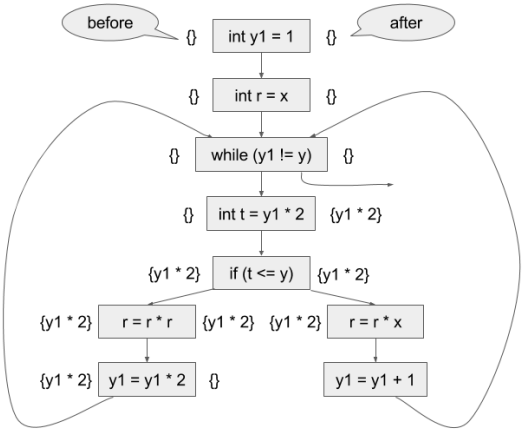

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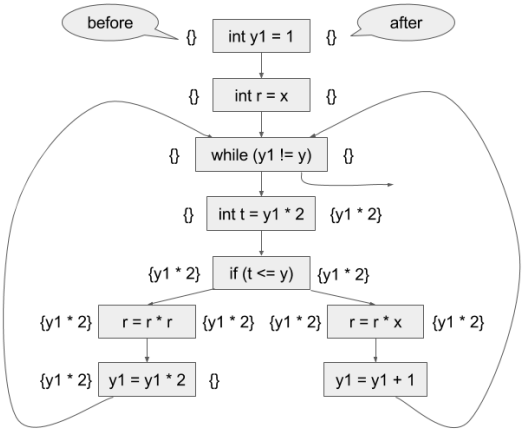

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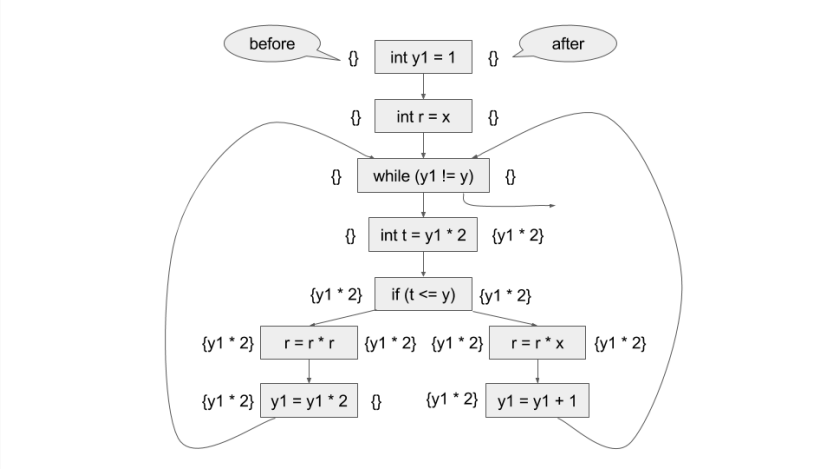

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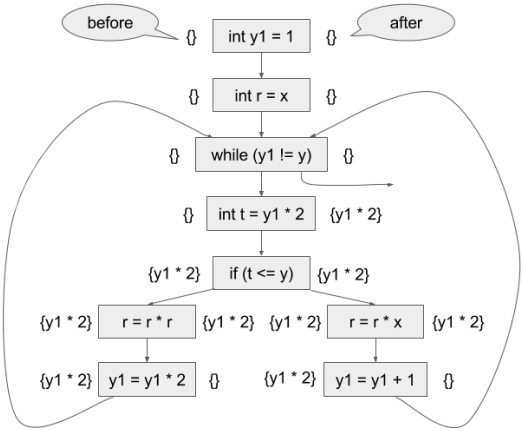
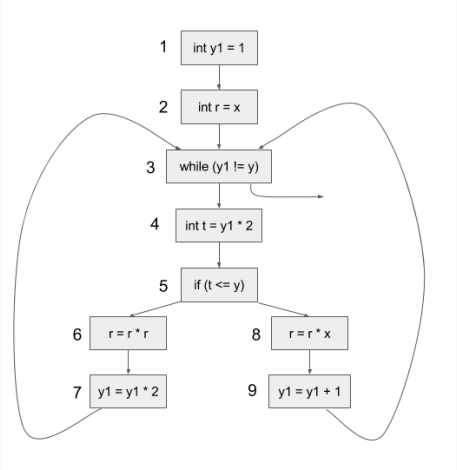


Formalizing The Analysis

How can these ideas be formalized?

1) introduce a variable i_n for the set of expressions available before node n, and a variable o_n for the set of expressions available after node n,
2) define equations between those variables,
3) solve those equations.

Equations



$$
\begin{aligned}
i_1 &= \{\} & o_1 &= i_1 \\
i_2 &= o_1 & o_2 &= i_2 \\
i_3 &= o_2 \cap o_7 \cap o_9 & o_3 &= i_3 \\
i_4 &= o_3 & o_4 &= \{\ \text{y1 * 2}\ \} \cup i_4 \\
i_5 &= o_4 & o_5 &= i_5 \\
i_6 &= o_5 & o_6 &= i_6 \downarrow \text{r} \\
i_7 &= o_6 & o_7 &= i_7 \downarrow \text{y1} \\
i_8 &= o_5 & o_8 &= i_8 \downarrow \text{r} \\
i_9 &= o_8 & o_9 &= i_9 \downarrow \text{y1}
\end{aligned}
$$

Notation: $S \downarrow x = S \setminus \{$ all expressions using x $\}$

Solving Equations

The equations can be solved by iteration:

- initialize all sets i_1, …, i_9, o_1, …, o_9 to the set of **all** nontrivial expressions in the program, here $\{$`y1*2, y1+1, r*r, r*x`$\}$,
- viewing the equations as assignments, compute the "new" value of those sets,
- iterate until a **fixed point** is reached.

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- iterate until a **fixed point** is reached.

Why initializing to all nontrivial expressions?

- Because we are interested in finding the largest sets satisfying the equations: the larger a set is, the more information it conveys (for this Analysis).

Solving Equations

To simplify the equations, we can first replace all i_k variables by their values, to obtain a simpler system, and then solve that system.

For our example, we obtain the following equations about the "out'' facts:

$$
\begin{aligned}
\mathsf{o}_1 &= \{\} & \mathsf{o}_6 &= \mathsf{o}_5 \downarrow \mathsf{r} \\
\mathsf{o}_2 &= \mathsf{o}_1 & \mathsf{o}_7 &= \mathsf{o}_6 \downarrow \mathsf{y1} \\
\mathsf{o}_3 &= \mathsf{o}_2 \cap \mathsf{o}_7 \cap \mathsf{o}_9 & \mathsf{o}_8 &= \mathsf{o}_5 \downarrow \mathsf{r} \\
\mathsf{o}_4 &= \mathsf{o}_3 \cup \{\ \mathsf{y1} * 2\ \} & \mathsf{o}_9 &= \mathsf{o}_8 \downarrow \mathsf{y1} \\
\mathsf{o}_5 &= \mathsf{o}_4
\end{aligned}
$$

Solving Equations

The simpler system can be solved by iterating until a fixed point is reached, which happens after 7 iterations.

it.	1	2	3	4	5	6	7
o_1	$\top$	{}	{}	{}	{}	{}	{}
o_2	$\top$	$\top$	{}	{}	{}	{}	{}
o_3	$\top$	$\top$	R	{}	{}	{}	{}
o_4	$\top$	$\top$	$\top$	{y1*2, r*r, r*x}	{y1*2}	{y1*2}	{y1*2}
o_5	$\top$	$\top$	$\top$	$\top$	{y1*2, r*r, r*x}	{y1*2}	{y1*2}
o_6	$\top$	Y	Y	Y	Y	{y1*2}	{y1*2}
o_7	$\top$	R	{}	{}	{}	{}	{}
o_8	$\top$	Y	Y	Y	Y	{y1*2}	{y1*2}
o_9	$\top$	R	{}	{}	{}	{}	{}

Notation: $Y = \{\texttt{y1*2},\ \texttt{y1+1}\}$, $R = \{\texttt{r*r},\ \texttt{r*x}\}$, $\top = Y \cup R$