Precise Reasoning with Structured Heaps and Collective Operations à la Map/Reduce

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Motivation

```
ListNode x = null; int i = 0
while (i < n) {
    ListNode y = new ListNode()
    y.tail = x
    y.head = i
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ListNode z = x; int sum = 0
while (z != null) {
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assert(sum == n*(n-1)/2)
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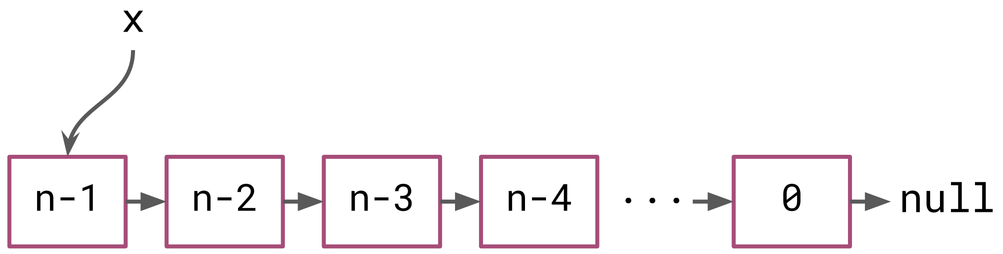

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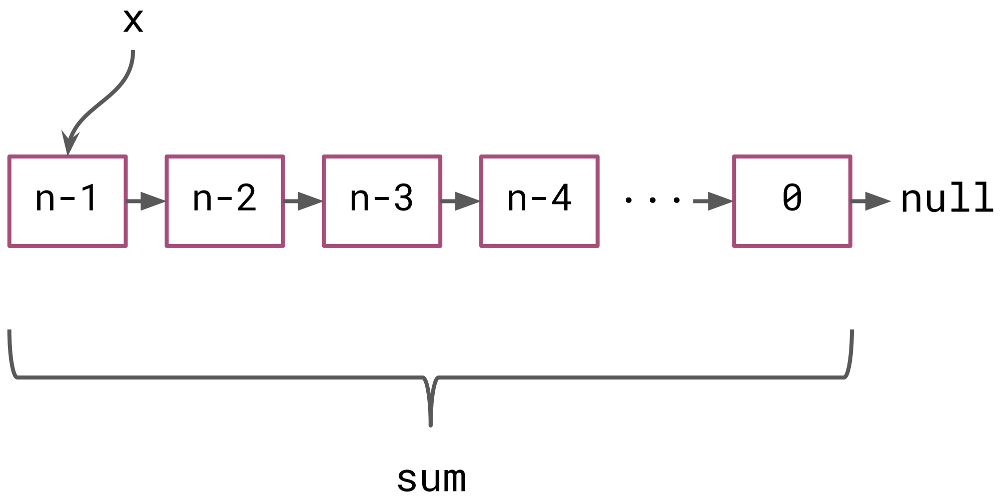

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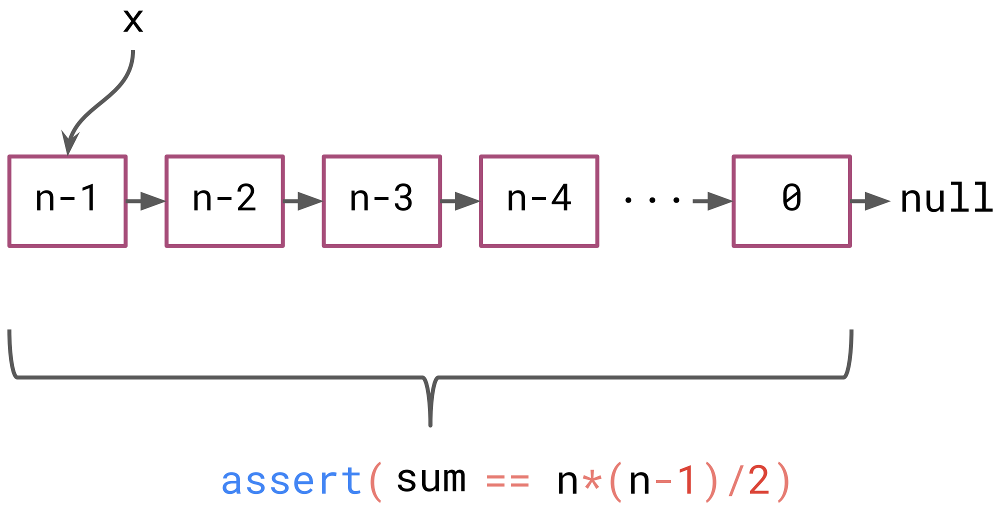

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- program abstractions are usually low-level *scalars*, rather than collections.
  e.g., a linked list contains natural numbers from `0` to `n-1`.

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Many techniques failed.

- program abstractions are usually low-level *scalars*, rather than collections.
  e.g., a linked list contains natural numbers from `0` to `n-1`.

- program abstractions lose the information of *time*.
  e.g., values at different loop iterations are not distinguished.

Our Solution

- Borrow ideas from Domain Specific Languages (DSL)
  - Translate low-level imperative program to high level functional program with semantics preserved
- First-class collective forms
  - The loop iteration index is not a free variable

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IMP:

int j = 0, sum = 0
while (j < k) {
    sum = sum + j
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```
IMP:                          FUN:

int j = 0, sum = 0            let j = λ(i).if (i > 0)
while (j < k) {                           then j(i-1)+1
    sum = sum + j   ──→                   else 1
    j = j + 1                 let sum = λ(i).if (i > 0)
}                                           then sum(i-1)+j(i-1)
                                            else 0
                              let n = #(i).!(j(i) <= k)
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while (j < k) {
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let j = λ(i).if (i > 0)
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```

STORE:

```
j→if (n > 0) then j(n-1)
              else 0
sum→if (n > 0) then sum(n-1)
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IMP:

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int j = 0, sum = 0
while (j < k) {
    sum = sum + j    ⟶
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let j = λ(i).if (i > 0)
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```
int j = 0, sum = 0
while (j < k) {
    sum = sum + j    →
    j = j + 1
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FUN:

```
let j = λ(i).Σ(i2 < i + 1).1

let sum = λ(i).if (i > 0)
                 then sum(i-1)+j(i-1)
                 else 0
let n = #(i).!(j(i) <= k)
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j→if (n > 0) then j(n-1)
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let j = λ(i).i + 1

let sum = λ(i).(i + 1) * i / 2

let n = #(i).!(j(i) <= k)
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STORE:

```
j→k

sum→(k - 1) * k / 2
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Collective Operations for Linked List

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IMP:

ListNode x = null; int i = 0
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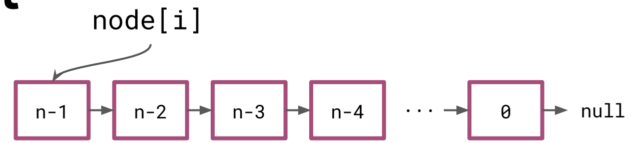

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let y = λ(i).if (i>0) then &new:node[i]
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let node = λ(i).
  newArray(i2<i).
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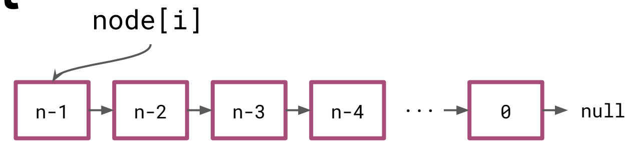

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let sum = λ(i).if (i>0) then
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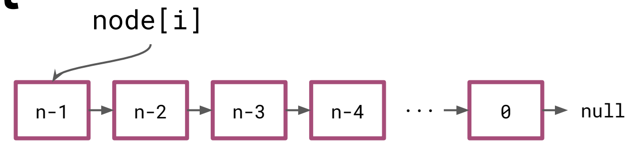

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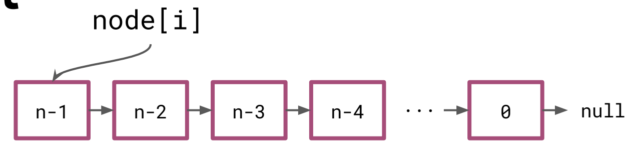

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let z = λ(i).if (i>0) then &new:node[n-i-2]
                      else &new:node[n-2]
let sum = λ(i).(i+1) * i / 2
```

Summary

- To verify program with loops, we translate low-level code to high-level DSL with collective forms
- The semantics and errors are preserved during translation
- Heap abstraction also use collective forms to reflect program structure
- We have scaled up our approach to a subset of C and use it to successfully verify programs from SV-COMP benchmarks

Thanks!